# 2008 Higher Mathematics
# Paper 1 Section A

| | |
|---|---|
| 1 | C |
| 2 | D |
| 3 | C |
| 4 | B |
| 5 | A |
| 6 | B |
| 7 | C |
| 8 | D |
| 9 | B |
| 10 | A |
| 11 | B |
| 12 | C |
| 13 | A |
| 14 | B |
| 15 | C |
| 16 | A |
| 17 | C |
| 18 | C |
| 19 | B |
| 20 | D |

# 1.21

| QU | part | mk | code | calc | source | ss | pd | ic | C | B | A | | U1 | U2 | U3 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1.21 | a | 6 | C8,C9 | NC | | 1 | 3 | 2 | 6 | | | | 6 | | |
| | b | 5 | A21,A22 | | | 1 | 3 | 1 | 5 | | | | | 5 | |
| | c | 4 | C10 | | | | | 4 | 2 | 2 | | | 4 | | |

A function $f$ is defined on the set of real numbers by $f(x) = x^3 - 3x + 2$.

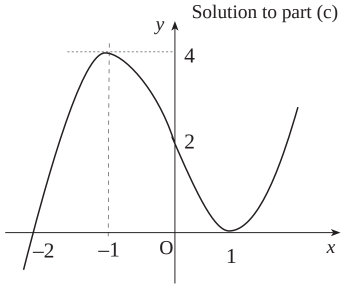
$(a)$ Find the coordinates of the stationary points on the curve $y = f(x)$ and determine their nature. 6

$(b)$ (i) Show that $(x-1)$ is a factor of $x^3 - 3x + 2$.

(ii) Hence or otherwise factorise $x^3 - 3x + 2$ fully. 5

$(c)$ State the coordinates of the points where the curve with equation $y = f(x)$ meets both the axes and hence sketch the curve. 4

Solution to part (c)




The primary method is based on this generic marking scheme which may be used as a guide for any method not shown in detail.

**Generic Marking Scheme**

- •1 ss set derivative to zero
- •2 pd differentiate
- •3 pd solve
- •4 pd evaluate $y$-coordinates
- •5 ic justification
- •6 ic state conclusions
- •7 ss know to use $x = 1$
- •8 pd complete eval. & conclusion
- •9 ic start to find quadratic factor
- •10 pd complete quadratic factor
- •11 pd factorise completely
- •12 ic interpret $y$-intercept
- •13 ic interpret $x$-intercepts
- •14 ic sketch : showing turning points
- •15 ic sketch : showing intercepts

**Primary Method : Give 1 mark for each •**

- •1 $f'(x) = 0$
- •2 $3x^2 - 3$
- •3 •4

$$\begin{array}{c|c|c|} & \bullet^3 & \bullet^4 \\ x & -1 & 1 \\ y & 4 & 0 \end{array}$$

- •5 •6

$$\begin{array}{c|ccc|ccc|} & & \bullet^5 & & & \bullet^6 & \\ & \dots & -1 & \dots & \dots & 1 & \dots \\ f' & + & 0 & - & - & 0 & + \\ & \text{max} & at\, x = & -1 & \text{min} & at\, x = & 1 \end{array}$$

- •7 know to use $x = 1$
- •8 $1 - 3 + 2 = 0 \Rightarrow x - 1$ is a factor
- •9 $(x-1)(x^2 ...)$
- •10 $(x-1)(x^2 + x - 2)$
- •11 $(x-1)(x-1)(x+2)$ ***stated explicitly***
- •12 $(0,2)$
- •13 $(-2,0)$, $(1,0)$
- •14 Sketch with turning pts marked
- •15 Sketch with $(0,2)$ or $(-2,0)$

**Notes**

1 The "=0" shown at •1 must appear at least once before the •3 stage.

2 An unsimplified $\sqrt{1}$ should be penalised at the first occurrence.

3 •3 is only available as a consequence of solving $f'(x) = 0$.

4 The nature table must reflect previous working from •3.

5 Candidates who introduce an extra solution at the •3 stage cannot earn •3.

6 The use of the 2nd derivative is an acceptable strategy for •5.

7 As shown in the Primary Method, (•3 and •4) and (•5 and •6) can be marked in series or in parallel.

8 The working for (b) may appear in (a) or vice versa. Full marks are available wherever the working occurs.

**Notes**

9 In Primary method •8 and alternative •9, candidates must show some acknowledgement of the resulting "0". Although a statement wrt the zero is preferable, accept something as simple as "underlining the zero".

**Alternative Method : •7 to •10**

•7

$$\begin{array}{c|cccc} 1 & 1 & 0 & -3 & 2 \\ \hline & & & & \end{array}$$

•8

$$\begin{array}{c|cccc} 1 & 1 & 0 & -3 & 2 \\ & & 1 & 1 & -2 \\ \hline & 1 & 1 & -2 & 0 \end{array}$$

•9 $f(1) = 0$ *so* $(x-1)$ *is a factor*

•10 $x^2 + x - 2$

**Notes**

10 Evidence for •12 and •13 may not appear until the sketch.

11 •14 and •15 are only available for the graph of a cubic.

**Nota Bene**

For candidates who omit the $x^2$ coeff. leading to

•7 X

•8 √

$$\begin{array}{c|ccc} 1 & 1 & -3 & 2 \\ & & 1 & -2 \\ \hline & 1 & -2 & 0 \end{array}$$

•9 √ $f(1) = 0$ *so* $(x-1)$.........

•10 X $x^2 - 2x$

•11 √ $x(x-1)(x-2)$

***but***

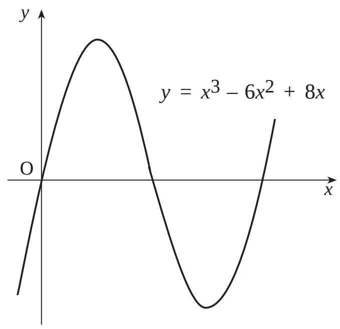
•10 X $x - 2$

•11 X $(x-1)(x-2)$

# 1.22

| qu | part | mk | code | calc | source | ss | pd | ic | C | B | A | | U1 | U2 | U3 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1.22 | a | 5 | C4 | NC | | 2 | 3 | | 5 | | | | 5 | | |
| | b | 2 | C11 | | | 1 | | 1 | 2 | | | | | 2 | |

The diagram shows a sketch of the curve with equation $y = x^3 - 6x^2 + 8x$.




$(a)$ Find the coordinates of the points on the curve where the gradient of the tangent is $-1$. 5

$(b)$ The line $y = 4 - x$ is a tangent to this curve at a point A. Find the coordinates of A. 2

The primary method is based on this generic marking scheme which may be used as a guide for any method not shown in detail.

**Generic Marking Scheme**

- $\bullet^1$ ss know to differentiate
- $\bullet^2$ pd differentiate
- $\bullet^3$ ss set derivative to $-1$
- $\bullet^4$ pd factorise and solve
- $\bullet^5$ pd solve for $y$
- $\bullet^6$ ss use gradient
- $\bullet^7$ ic interpret result

**Primary Method : Give 1 mark for each •**

- $\bullet^1$ $\frac{dy}{dx} = ..(1\ \textit{term correct})$ *s / i by* $\bullet^2$
- $\bullet^2$ $3x^2 - 12x + 8$ *s / i by* $\bullet^3$
- $\bullet^3$ $3x^2 - 12x + 8 = -1$

| | | $\bullet^4$ | $\bullet^5$ |
|---|---|---|---|
| $\bullet^4$ | $x$ | 1 | 3 |
| $\bullet^5$ | $y$ | 3 | $-3$ |

- $\bullet^6$ $y = 4 - x$ has gradient $= -1$
- $\bullet^7$ *check* $(3,-3)$ and reject
  *check* $(1,3)$ and accept

**Notes**

1 *in* $(a)$

$\bullet^1$ $\surd$ $\frac{dy}{dx} = ..(1\ \textit{term correct})$

$\bullet^2$ $\surd$ $3x^2 - 12x + 8$

For candidates who now guess $x = 1$ and check that $\frac{dy}{dx} = -1$, only one further mark ($\bullet^3$) can be awarded. Guessing and checking further answers gains no more credit.

2 An "=0" must appear at least once in the two lines shown in the alternative for $\bullet^6$ and $\bullet^7$.

**Common Error**

- $\bullet^1$ $\surd$ $\frac{dy}{dx} = ..(1\ \textit{term correct})$
- $\bullet^2$ $\surd$ $3x^2 - 12x + 8$
- $\bullet^3$ $X$ $3x^2 - 12x + 8 = 0$
- $\bullet^4$ $X$ irrespective of what is written.
- $\bullet^5$ $X$

**Alternative for $\bullet^6$ and $\bullet^7$**

$\bullet^6$ $\begin{cases} x^3 - 6x^2 + 8x = 4 - x \\ x^3 - 6x^2 + 9x - 4 = 0 \\ (x-1)(x^2 - 5x + 4) \\ \quad (x-4)(x-1) \end{cases}$

$\bullet^7$ $\begin{cases} \text{repeated root implies} \\ \text{tangent at } (1,3). \end{cases}$

# 1.23

| qu | part | mk | A3 | calc | source | ss | pd | ic | C | B | A | | U1 | U2 | U3 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1.23 | a | 3 | A4 | NC | | | | 3 | 3 | | | | 3 | | |
| | b | 5 | A31 | | | 2 | 2 | 1 | | 1 | 4 | | | | 5 |

Functions $f, g$ and $h$ are defined on suitable domains by $f(x) = x^2 - x + 10$, $g(x) = 5 - x$ and $h(x) = \log_2 x$.

$(a)$ Find expressions for $h\big(f(x)\big)$ and $h\big(g(x)\big)$. 3

$(b)$ Hence solve $h\big(f(x)\big) - h\big(g(x)\big) = 3$ 5

The primary method is based on this generic marking scheme which may be used as a guide for any method not shown in detail.

**Generic Marking Scheme**

| | | |
|---|---|---|
| $\bullet^1$ | ic | interpretation composition |
| $\bullet^2$ | ic | interpretation composition |
| $\bullet^3$ | ic | interpretation composition |
| $\bullet^4$ | ss | use log laws |
| $\bullet^5$ | ss | convert to exponential form |
| $\bullet^6$ | pd | process conversion |
| $\bullet^7$ | pd | express in standard form |
| $\bullet^8$ | ic | find valid solutions |

**Primary Method : Give 1 mark for each •**

| | |
|---|---|
| $\bullet^1$ | $h\big(f(x)\big) = h\big(x^2 - x + 10\big)$ $s\ /\ i\ by\ \bullet^2$ |
| $\bullet^2$ | $\log_2\left(x^2 - x + 10\right)$ |
| $\bullet^3$ | $\log_2(5 - x)$ |
| $\bullet^4$ | $\log_2\left(\dfrac{x^2 - x + 10}{5 - x}\right)$ |
| $\bullet^5$ | $\dfrac{x^2 - x + 10}{5 - x} = 2^3$ |
| $\bullet^6$ | $x^2 - x + 10 = 8(5 - x)$ |
| $\bullet^7$ | $x^2 + 7x - 30 = 0$ |
| $\bullet^8$ | $x = 3,\ -10$ |

**Notes**

1 In (a) 2 marks are available for finding one of $h(f(x))$ or $h(g(x))$ and the third mark is for the other.

2 Treat $\log_2 x^2 - x + 10$ *and* $\log_2 5 - x$ as bad form.

3 The omission of the base should not be penalised in $\bullet^2$ to $\bullet^4$.

4 $\bullet^7$ is only available for a quadratic equation and $\bullet^8$ must be the follow-through solutions.

**Common Error 1**

| | | |
|---|---|---|
| $\bullet^4$ | $X$ | $\log_2(x^2 + 5) = 3$ |
| $\bullet^5$ | $\surd$ | $x^2 + 5 = 2^3$ |
| $\bullet^6$ | $X$ | $x^2 = 3$ |
| $\bullet^7$ | $X$ | $x = \pm\sqrt{3}$ |
| $\bullet^8$ | $X$ | not available |

**Common Error 2**

| | | |
|---|---|---|
| $\bullet^4$ | $\surd$ | $\log_2\left(\dfrac{x^2 - x + 10}{5 - x}\right)$ |
| | | $\log_2\left(\dfrac{x^2 \cancel{- x} + \cancel{10}^{2}}{\cancel{5}^{1} \cancel{- x}}\right)$ |
| | | $\log_2\left(x^2 + 2\right) = 3$ |
| $\bullet^5$ | $X\surd$ | $x^2 + 2 = 2^3$ |
| $\bullet^6$ | $X$ | $x = \pm\sqrt{6}$ |
| $\bullet^7$ | $X$ | *not available* |
| $\bullet^8$ | $X$ | *not available* |

**Common Error 3**

| | | |
|---|---|---|
| $\bullet^4$ | $X$ | not available |
| $\bullet^5$ | $\surd$ | $\log_2(x^2 - x + 10) - \log_2(5 - x) = \log_2 8$ |
| $\bullet^6$ | $X$ | $x^2 - x + 10 - (5 - x) = 8$ |
| $\bullet^7$ | $X$ | not available |
| $\bullet^8$ | $X$ | not available |