

X100/303

NATIONAL
QUALIFICATIONS
2006

FRIDAY, 19 MAY
10.30 AM – 12.00 NOON

MATHEMATICS HIGHER

Units 1, 2 and 3
Paper 2

Read Carefully

- 1 Calculators may be used in this paper.
- 2 Full credit will be given only where the solution contains appropriate working.
- 3 Answers obtained by readings from scale drawings will not receive any credit.

$f(x)$	$\int f(x) dx$
$\sin ax$	$-\frac{1}{a} \cos ax + C$
$\cos ax$	$\frac{1}{a} \sin ax + C$

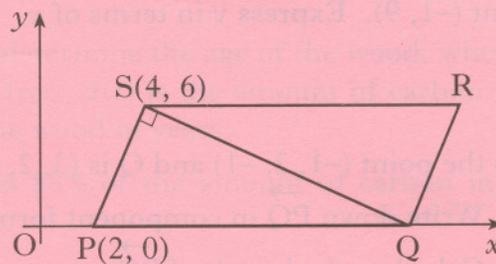


ALL questions should be attempted.

Marks

1. PQRS is a parallelogram. P is the point (2, 0), S is (4, 6) and Q lies on the x -axis, as shown.

The diagonal QS is perpendicular to the side PS.

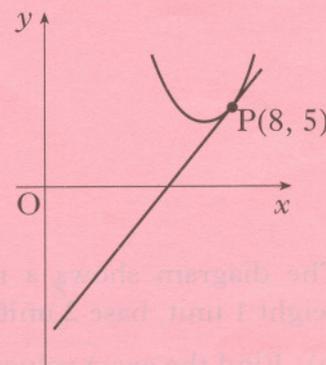


- (a) Show that the equation of QS is $x + 3y = 22$. 4
 (b) Hence find the coordinates of Q and R. 2

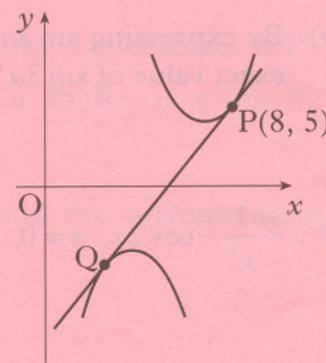
2. Find the value of k such that the equation $kx^2 + kx + 6 = 0$, $k \neq 0$, has equal roots. 4

3. The parabola with equation $y = x^2 - 14x + 53$ has a tangent at the point P(8, 5).

- (a) Find the equation of this tangent. 4



- (b) Show that the tangent found in (a) is also a tangent to the parabola with equation $y = -x^2 + 10x - 27$ and find the coordinates of the point of contact Q. 5



4. The circles with equations $(x - 3)^2 + (y - 4)^2 = 25$ and $x^2 + y^2 - kx - 8y - 2k = 0$ have the same centre.

Determine the radius of the larger circle. 5

5. The curve $y = f(x)$ is such that $\frac{dy}{dx} = 4x - 6x^2$. The curve passes through the point $(-1, 9)$. Express y in terms of x . 4

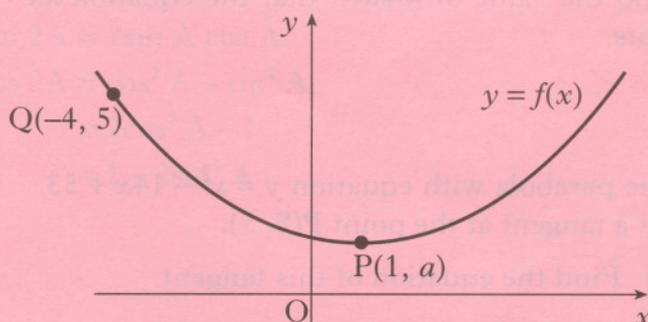
6. P is the point $(-1, 2, -1)$ and Q is $(3, 2, -4)$.

- (a) Write down \vec{PQ} in component form. 1
 (b) Calculate the length of \vec{PQ} . 1
 (c) Find the components of a unit vector which is parallel to \vec{PQ} . 1

7. The diagram shows the graph of a function $y = f(x)$.

Copy the diagram and on it sketch the graphs of:

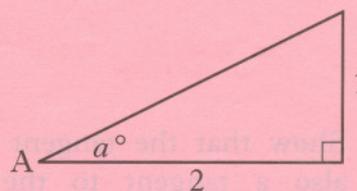
- (a) $y = f(x - 4)$; 2
 (b) $y = 2 + f(x - 4)$. 2



8. The diagram shows a right-angled triangle with height 1 unit, base 2 units and an angle of a° at A.

(a) Find the exact values of:

- (i) $\sin a^\circ$;
 (ii) $\sin 2a^\circ$.



(b) By expressing $\sin 3a^\circ$ as $\sin(2a + a)^\circ$, find the exact value of $\sin 3a^\circ$. 4

9. If $y = \frac{1}{x^3} - \cos 2x$, $x \neq 0$, find $\frac{dy}{dx}$. 4

10. A curve has equation $y = 7\sin x - 24\cos x$.

- (a) Express $7\sin x - 24\cos x$ in the form $k\sin(x - a)$ where $k > 0$ and $0 \leq a \leq \frac{\pi}{2}$. 4
 (b) Hence find, in the interval $0 \leq x \leq \pi$, the x -coordinate of the point on the curve where the gradient is 1. 3

11. It is claimed that a wheel is made from wood which is over 1000 years old.

To test this claim, carbon dating is used.

The formula $A(t) = A_0 e^{-0.000124t}$ is used to determine the age of the wood, where A_0 is the amount of carbon in any living tree, $A(t)$ is the amount of carbon in the wood being dated and t is the age of the wood in years.

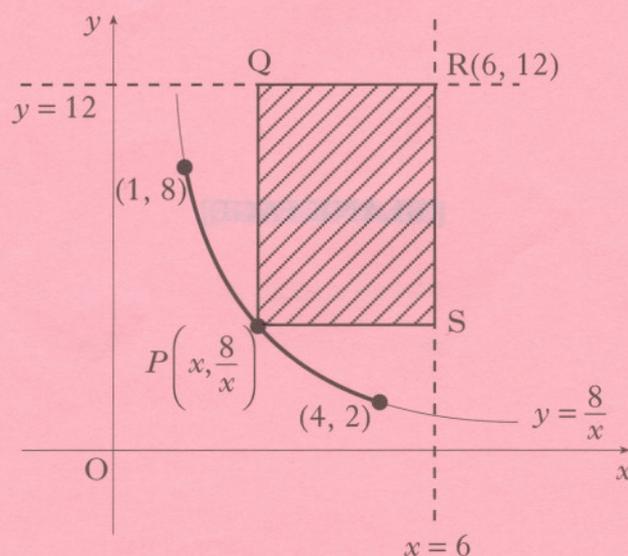
For the wheel it was found that $A(t)$ was 88% of the amount of carbon in a living tree.

Is the claim true?

5

12. PQRS is a rectangle formed according to the following conditions:

- it is bounded by the lines $x = 6$ and $y = 12$
- P lies on the curve with equation $y = \frac{8}{x}$ between (1, 8) and (4, 2)
- R is the point (6, 12).



- (a) (i) Express the lengths of PS and RS in terms of x , the x -coordinate of P.
 (ii) Hence show that the area, A square units, of PQRS is given by

$$A = 80 - 12x - \frac{48}{x}.$$

3

- (b) Find the greatest and least possible values of A and the corresponding values of x for which they occur.

8

[END OF QUESTION PAPER]