

X100/301

NATIONAL
QUALIFICATIONS
2005

FRIDAY, 20 MAY
9.00 AM – 10.10 AM

MATHEMATICS HIGHER

Units 1, 2 and 3

Paper 1

(Non-calculator)

Read Carefully

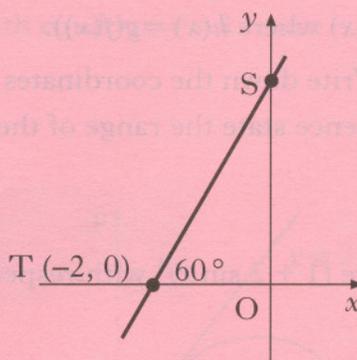
- 1 Calculators may **NOT** be used in this paper.
- 2 Full credit will be given only where the solution contains appropriate working.
- 3 Answers obtained by readings from scale drawings will not receive any credit.



ALL questions should be attempted.

Marks

1. Find the equation of the line ST, where T is the point $(-2, 0)$ and angle STO is 60° .



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2. Two congruent circles, with centres A and B, touch at P.

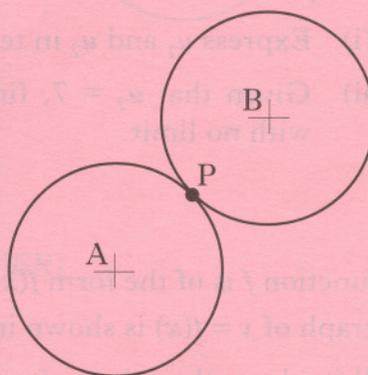
Relative to suitable axes, their equations are

$$x^2 + y^2 + 6x + 4y - 12 = 0 \text{ and}$$

$$x^2 + y^2 - 6x - 12y + 20 = 0.$$

(a) Find the coordinates of P.

(b) Find the length of AB.



3

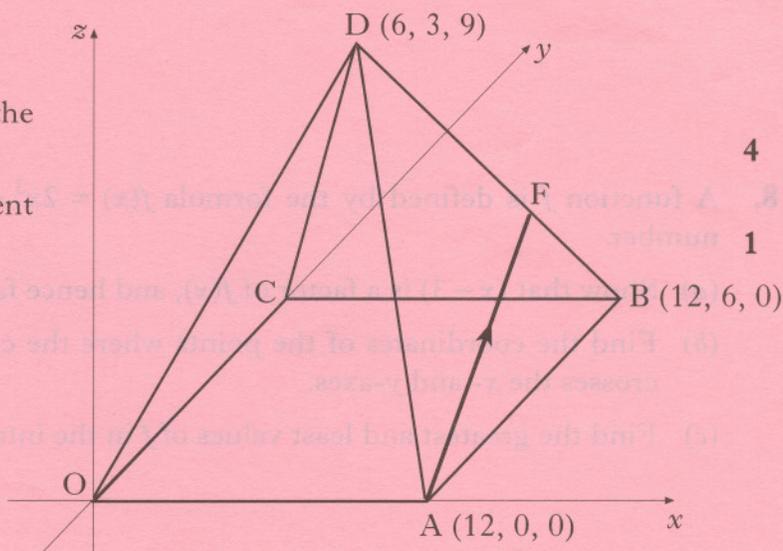
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3. D,OABC is a pyramid. A is the point $(12, 0, 0)$, B is $(12, 6, 0)$ and D is $(6, 3, 9)$.

F divides DB in the ratio 2:1.

(a) Find the coordinates of the point F.

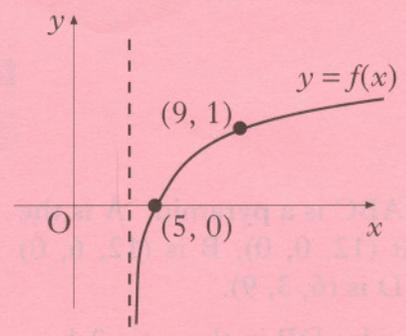
(b) Express \vec{AF} in component form.



4

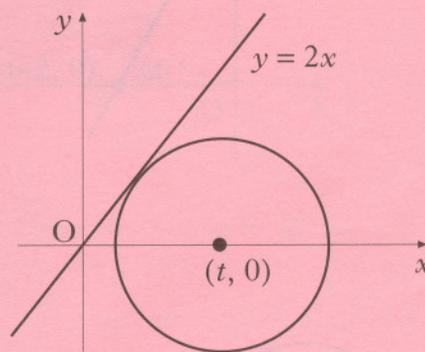
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[Turn over

4. Functions $f(x) = 3x - 1$ and $g(x) = x^2 + 7$ are defined on the set of real numbers.
- (a) Find $h(x)$ where $h(x) = g(f(x))$. 2
- (b) (i) Write down the coordinates of the minimum turning point of $y = h(x)$.
 (ii) Hence state the range of the function h . 2
5. Differentiate $(1 + 2 \sin x)^4$ with respect to x . 2
6. (a) The terms of a sequence satisfy $u_{n+1} = ku_n + 5$. Find the value of k which produces a sequence with a limit of 4. 2
- (b) A sequence satisfies the recurrence relation $u_{n+1} = mu_n + 5$, $u_0 = 3$.
- (i) Express u_1 and u_2 in terms of m .
 (ii) Given that $u_2 = 7$, find the value of m which produces a sequence with no limit. 5
7. The function f is of the form $f(x) = \log_b(x - a)$.
 The graph of $y = f(x)$ is shown in the diagram.
- (a) Write down the values of a and b . 2
- (b) State the domain of f . 1
- 
8. A function f is defined by the formula $f(x) = 2x^3 - 7x^2 + 9$ where x is a real number.
- (a) Show that $(x - 3)$ is a factor of $f(x)$, and hence factorise $f(x)$ fully. 5
- (b) Find the coordinates of the points where the curve with equation $y = f(x)$ crosses the x - and y -axes. 2
- (c) Find the greatest and least values of f in the interval $-2 \leq x \leq 2$. 5
9. If $\cos 2x = \frac{7}{25}$ and $0 < x < \frac{\pi}{2}$, find the exact values of $\cos x$ and $\sin x$. 4

10. (a) Express $\sin x - \sqrt{3} \cos x$ in the form $k \sin(x - a)$ where $k > 0$ and $0 \leq a \leq 2\pi$. 4
 (b) Hence, or otherwise, sketch the curve with equation $y = 3 + \sin x - \sqrt{3} \cos x$ in the interval $0 \leq x \leq 2\pi$. 5

11. (a) A circle has centre $(t, 0)$, $t > 0$, and radius 2 units.
 Write down the equation of the circle.
 (b) Find the exact value of t such that the line $y = 2x$ is a tangent to the circle.



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