SCOTTISH CERTIFICATE OF EDUCATION 1997

THURSDAY, 8 MAY
1.00 PM – 3.30 PM

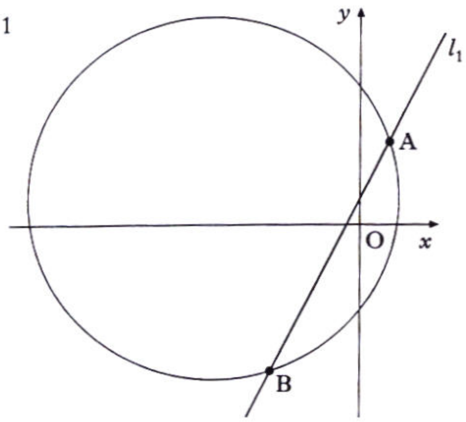
MATHEMATICS
HIGHER GRADE
Paper II

All questions should be attempted *Marks*

1. Diagram 1 shows a circle with equation $x^2 + y^2 + 10x - 2y - 14 = 0$ and a straight line, l_1, with equation $y = 2x + 1$.

   The line intersects the circle at A and B.

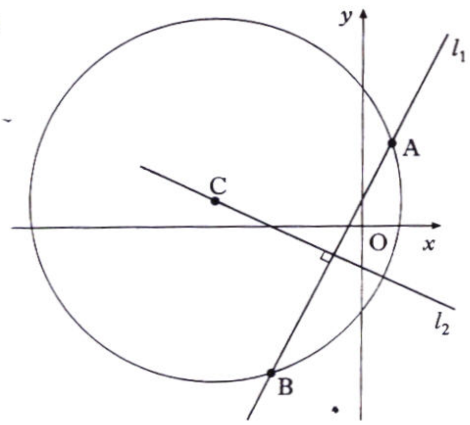
Diagram 1



(a) Find the coordinates of the points A and B. **(5)**

(b) Diagram 2 shows a second line, l_2, which passes through the centre of the circle, C, and is at right angles to line l_1.

Diagram 2



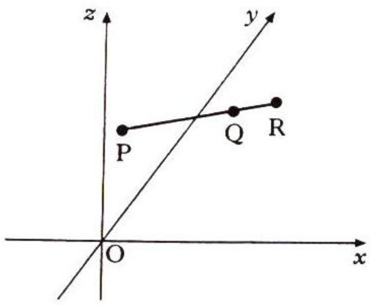
(i) Write down the coordinates of C. **(1)**

(ii) Find the equation of the line l_2. **(3)**

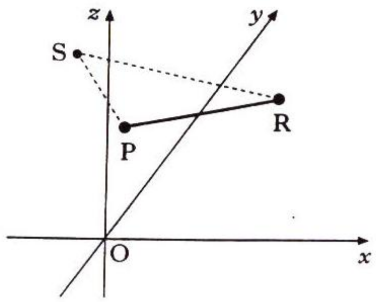
Marks

2. Relative to the axes shown and with an appropriate scale, P(–1, 3, 2) and Q(5, 0, 5) represent points on a road. The road is then extended to the point R such that $\overrightarrow{PR} = \frac{4}{3}\overrightarrow{PQ}$.

(*a*) Find the coordinates of R. **(3)**



(*b*) Roads from P and R are built to meet at the point S (–2, 2, 5). Calculate the size of angle PSR. **(7)**



3. The sum of £1000 is placed in an investment account on January 1st and, thereafter, £100 is placed in the account on the first day of each month.

- Interest at the rate of 0·5% per month is credited to the account on the last day of each month.
- This interest is calculated on the amount in the account on the first day of the month.

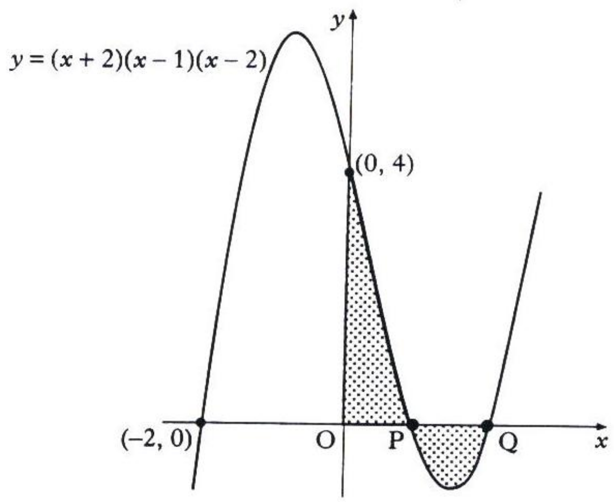
(*a*) How much is in the account on June 30th? **(4)**

(*b*) On what date does the account first exceed £2000? **(2)**

(*c*) Find a recurrence relation which describes the amount in the account, explaining your notation carefully. **(3)**

Marks

4. The diagram shows a sketch of the graph of $y = (x + 2)(x - 1)(x - 2)$. The graph cuts the axes at $(-2, 0)$, $(0, 4)$ and the points P and Q.



(*a*) Write down the coordinates of P and Q. **(2)**

(*b*) Find the total shaded area. **(7)**

Marks

5. Diagram 1 shows a sketch of part of the graph of $y = f(x)$ where $f(x) = (x - 2)^2 + 1$.

The graph cuts the y-axis at A and has a minimum turning point at B.

(*a*) Write down the coordinates of A and B. **(3)**

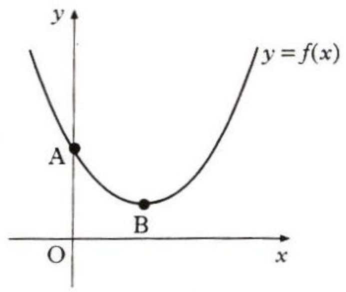


Diagram 1

(*b*) Diagram 2 shows the graphs of $y = f(x)$ and $y = g(x)$ where $g(x) = 5 + 4x - x^2$.

Find the area enclosed by the two curves. **(5)**

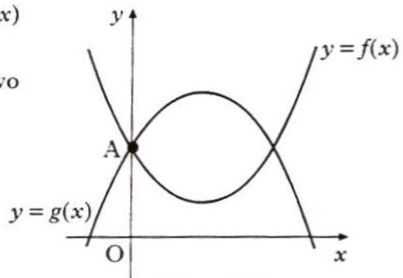


Diagram 2

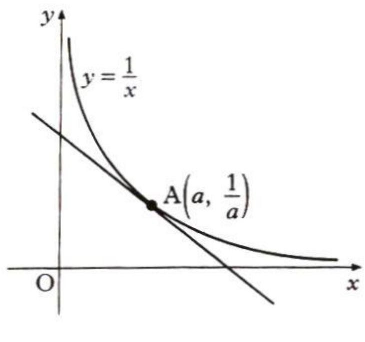
(*c*) $g(x)$ can be written in the form $m + n \times f(x)$ where m and n are constants.

Write down the values of m and n. **(2)**

6. (*a*) A sketch of part of the graph of $y = \frac{1}{x}$ is shown in the diagram.

The tangent at $A\left(a, \frac{1}{a}\right)$ has been drawn.

Find the gradient of this tangent. **(4)**



(*b*) Hence show that the equation of this tangent is $x + a^2y = 2a$. **(2)**

(*c*) This tangent cuts the y-axis at B and the x-axis at C.

(i) Calculate the area of triangle OBC. **(3)**

(ii) Comment on your answer to c(i). **(1)**

Marks

7. In certain topics in Mathematics, such as calculus, we often require to write an expression such as $\dfrac{8x+1}{(2x+1)(x-1)}$ in the form $\dfrac{2}{2x+1}+\dfrac{3}{x-1}$.

$\dfrac{2}{2x+1}+\dfrac{3}{x-1}$ are called **Partial Fractions** for $\dfrac{8x+1}{(2x+1)(x-1)}$.

The worked example shows you how to find partial fractions for the expression $\dfrac{6x+2}{(x+2)(x-3)}$.

Worked Example

Find partial fractions for $\dfrac{6x+2}{(x+2)(x-3)}$.

Let $\dfrac{6x+2}{(x+2)(x-3)} = \dfrac{A}{x+2}+\dfrac{B}{x-3}$ where A and B are constants

$$= \frac{A(x-3)}{(x+2)(x-3)}+\frac{B(x+2)}{(x-3)(x+2)}$$

ie $\dfrac{6x+2}{(x+2)(x-3)} = \dfrac{A(x-3)+B(x+2)}{(x+2)(x-3)}$

Hence $6x+2 = A(x-3)+B(x+2)$ for all values of x.

A and B can be found as follows:

Select a value of x that makes the first bracket zero

Let $x = 3$ (this eliminates A)

$18+2 = A\times 0 + B\times 5$

$20 = 5B$

$\underline{B = 4}$

Select a value of x that makes the second bracket zero

Let $x = -2$ (this eliminates B)

$-12+2 = A\times(-5) + B\times 0$

$-10 = -5A$

$\underline{A = 2}$

Therefore $\dfrac{6x+2}{(x+2)(x-3)} = \dfrac{2}{x+2}+\dfrac{4}{x-3}$.

Find partial fractions for $\dfrac{5x+1}{(x-4)(x+3)}$. **(6)**

Marks

8. The radioactive element carbon-14 is sometimes used to estimate the age of organic remains such as bones, charcoal and seeds.

Carbon-14 decays according to a law of the form $y = y_0e^{kt}$ where y is the amount of radioactive nuclei present at time t years and y_0 is the initial amount of radioactive nuclei.

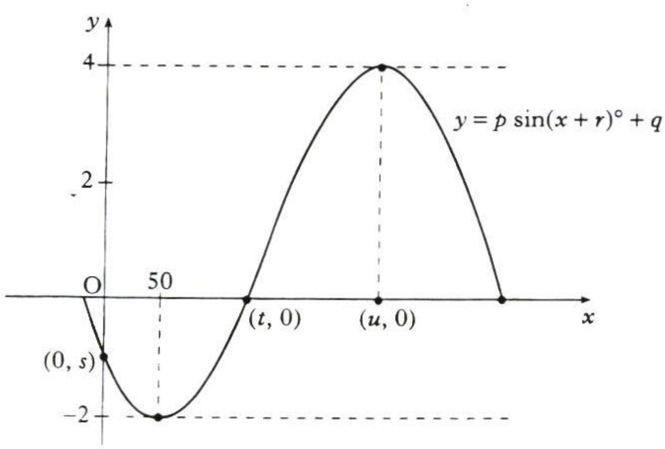
(*a*) The half-life of carbon-14, ie the time taken for half the radioactive nuclei to decay, is 5700 years. Find the value of the constant k, correct to 3 significant figures. **(3)**

(*b*) What percentage of the carbon-14 in a sample of charcoal will remain after 1000 years? **(3)**

9. The sketch represents part of the graph of a trigonometric function of the form $y = p\sin(x + r)° + q$. It crosses the axes at $(0, s)$ and $(t, 0)$, and has turning points at $(50, -2)$ and $(u, 4)$.

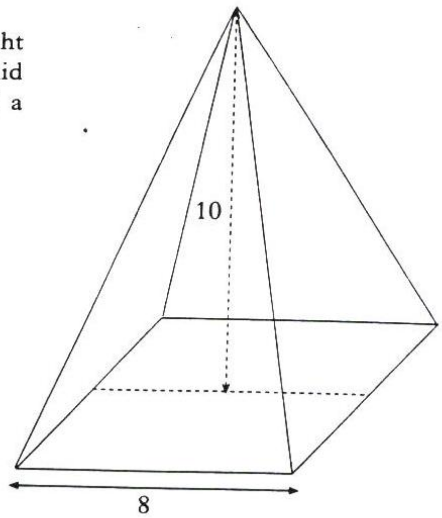
(*a*) Write down values for p, q, r and u. **(4)**

(*b*) Find the values for s and t. **(4)**

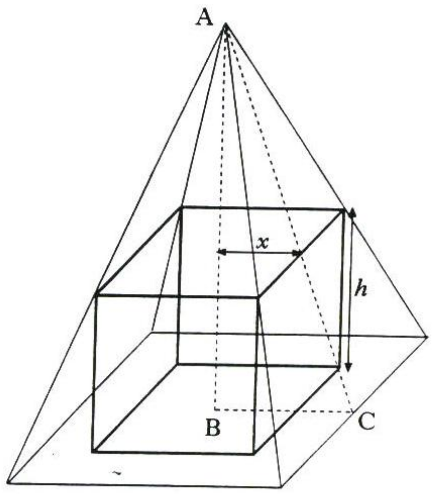


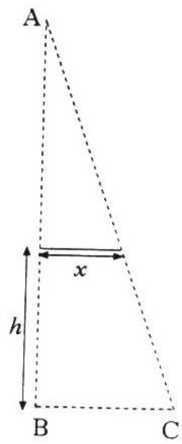
10. A cuboid is to be cut out of a right square-based pyramid. The pyramid has a square base of side 8 cm and a vertical height of 10 cm.

Marks



(*a*) The cuboid has a square base of side $2x$ cm and a height of h cm.





If the cuboid is to fit into the pyramid, use the information shown in triangle ABC, or otherwise, to show that:

(i) $h = 10 - \frac{5}{2}x$; **(3)**

(ii) the volume, V, of the cuboid is given by $V = 40x^2 - 10x^3$. **(1)**

(*b*) Hence, find the dimensions of the square-based cuboid with the greatest volume which can be cut from the pyramid. **(6)**

Marks

11. Two identical coins, radius 1 unit, are supported by horizontal and vertical plates at B and C. Diagram 1 shows the coins touching each other and the line of centres is inclined at p radians to the vertical.

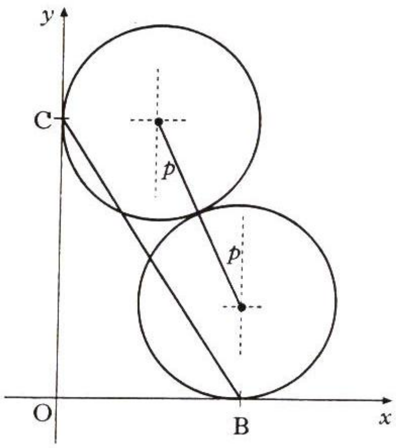


Diagram 1

Let d be the length of BC.

(*a*) (i) Show that $OB = 1 + 2\sin p$. **(1)**

(ii) Write down a similar expression for OC and hence show that $d^2 = 6 + 4\cos p + 4\sin p$. **(2)**

(*b*) (i) Express d^2 in the form $6 + k\cos(p - \alpha)$. **(4)**

(ii) Hence, write down the exact maximum value of d^2 and the value of p for which this occurs. **(2)**

(*c*) Diagram 2 shows the special case where $p = \frac{\pi}{4}$.

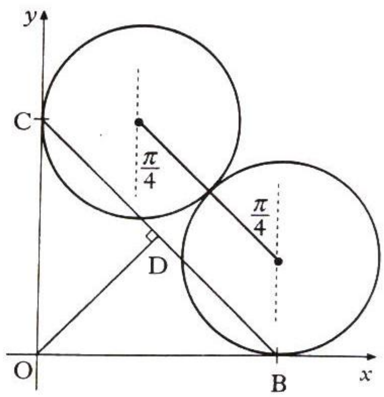


Diagram 2

(i) Show that $OB = 1 + \sqrt{2}$ and find the exact length of BD. **(2)**

(ii) Using your answer to (*b*) (ii), find the exact value of $\sqrt{6 + 4\sqrt{2}}$. **(2)**

[*END OF QUESTION PAPER*]