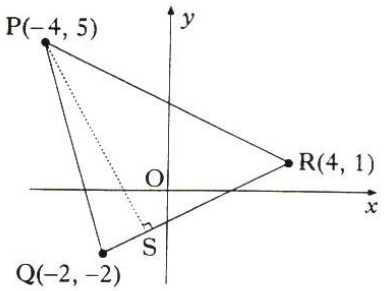
SCOTTISH CERTIFICATE OF EDUCATION 1997

THURSDAY, 8 MAY 9.30 AM – 11.30 AM

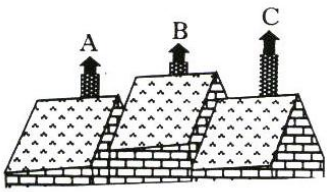
MATHEMATICS HIGHER GRADE
Paper I

All questions should be attempted *Marks*

NC 1. P(–4, 5), Q(–2, –2) and R(4, 1) are the vertices of triangle PQR as shown in the diagram. Find the equation of PS, the altitude from P. **(3)**



NC 2. Relative to a suitable set of axes, the tops of three chimneys have coordinates given by A(1, 3, 2), B(2, –1, 4) and C(4, –9, 8).
Show that A, B and C are collinear. **(3)**



NC 3. Functions f and g, defined on suitable domains, are given by $f(x) = 2x$ and $g(x) = \sin x + \cos x$.
Find $f(g(x))$ and $g(f(x))$. **(4)**

NC 4. The position vectors of the points P and Q are $\boldsymbol{p} = -\boldsymbol{i} + 3\boldsymbol{j} + 4\boldsymbol{k}$ and $\boldsymbol{q} = 7\boldsymbol{i} - \boldsymbol{j} + 5\boldsymbol{k}$ respectively.

(*a*) Express $\overrightarrow{PQ}$ in component form. **(2)**

(*b*) Find the length of PQ. **(1)**

NC 5. (*a*) Find a real root of the equation $2x^3 - 3x^2 + 2x - 8 = 0$. **(2)**

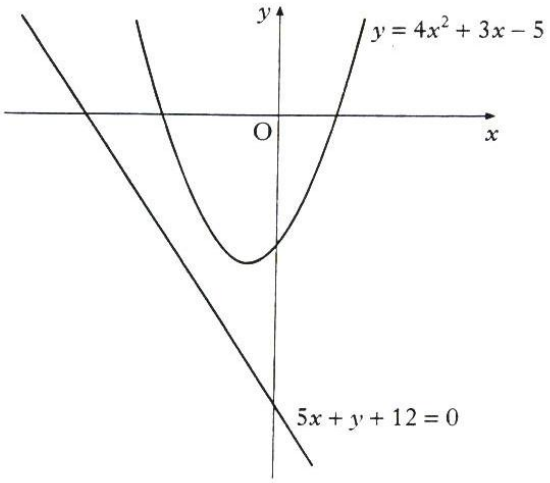
(*b*) Show algebraically that there are no other real roots. **(3)**

Marks

NC **6.** The diagram below shows a parabola with equation $y = 4x^2 + 3x - 5$ and a straight line with equation $5x + y + 12 = 0$.

A tangent to the parabola is drawn parallel to the given straight line.

Find the x-coordinate of the point of contact of this tangent. **(5)**



NC **7.** If $x°$ is an acute angle such that $\tan x° = \frac{4}{3}$, show that the exact value of $\sin(x+30)°$ is $\frac{4\sqrt{3}+3}{10}$. **(3)**

NC **8.** Given that $y = 2x^2 + x$, find $\frac{dy}{dx}$ and hence show that $x\left(1+\frac{dy}{dx}\right) = 2y$. **(3)**

NC **9.** (*a*) Show that the function $f(x) = 2x^2 + 8x - 3$ can be written in the form $f(x) = a(x + b)^2 + c$ where a, b and c are constants. **(3)**

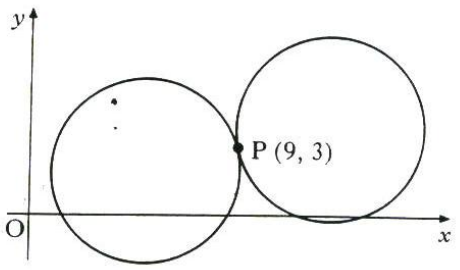
(*b*) Hence, or otherwise, find the coordinates of the turning point of the function f. **(1)**

NC **10.** Find the value of $\int_1^4 \sqrt{x}\,dx$. **(4)**

11. Express $2\sin x° - 5\cos x°$ in the form $k\sin(x - \alpha)°$, $0 \leq \alpha < 360$ and $k > 0$. **(4)**

12. Two identical circles touch at the point P (9, 3) as shown in the diagram. One of the circles has equation $x^2 + y^2 - 10x - 4y + 12 = 0$.

Find the equation of the other circle. **(5)**

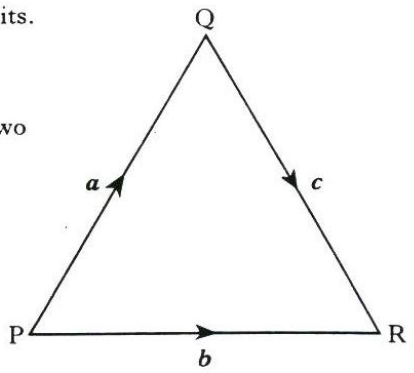


Marks

13. PQR is an equilateral triangle of side 2 units.

$\overrightarrow{PQ} = \boldsymbol{a}$, $\overrightarrow{PR} = \boldsymbol{b}$ and $\overrightarrow{QR} = \boldsymbol{c}$.

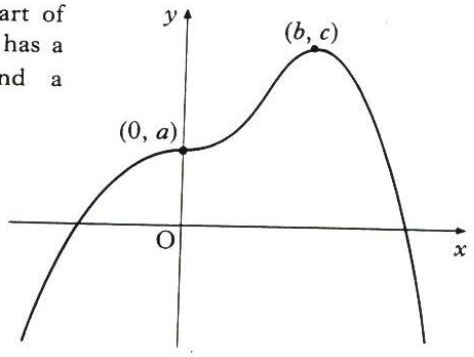
Evaluate $\boldsymbol{a}.(\boldsymbol{b} + \boldsymbol{c})$ and hence identify two vectors which are perpendicular. **(4)**



14. For what range of values of c does the equation $x^2 + y^2 - 6x + 4y + c = 0$ represent a circle? **(3)**

15. The curve $y = f(x)$ passes through the point $\left(\frac{\pi}{12}, 1\right)$ and $f'(x) = \cos 2x$.
Find $f(x)$. **(3)**

16. The diagram shows a sketch of part of the graph of $y = f(x)$. The graph has a point of inflection at $(0, a)$ and a maximum turning point at (b, c).

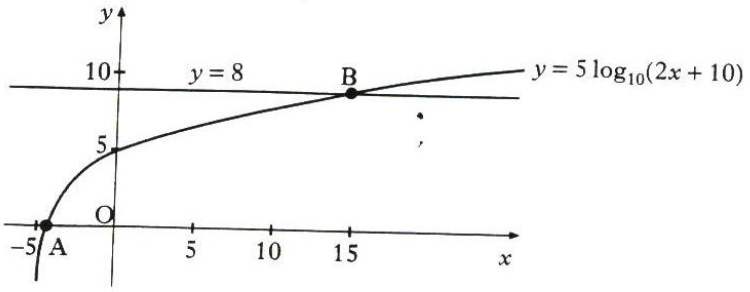


(a) Make a copy of this diagram and on it sketch the graph of $y = g(x)$ where $g(x) = f(x) + 1$. **(2)**

(b) On a separate diagram, sketch the graph of $y = f'(x)$. **(2)**

(c) Describe how the graph of $y = g'(x)$ is related to the graph of $y = f'(x)$. **(1)**

17. Part of the graph of $y = 5\log_{10}(2x + 10)$ is shown in the diagram. This graph crosses the x-axis at the point A and the straight line $y = 8$ at the point B.
Find algebraically the x-coordinates of A and B. **(4)**



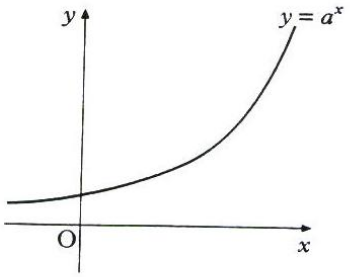
Marks

18. (*a*) Show that $2\cos 2x° - \cos^2 x° = 1 - 3\sin^2 x°$. **(2)**

(*b*) **Hence** solve the equation

$2\cos 2x° - \cos^2 x° = 2\sin x°$ in the interval $0 \leq x < 360$. **(4)**

19. The diagram shows a sketch of part of the graph of $y = a^x$, $a > 1$.



(*a*) If $(1, t)$ and $(u, 1)$ lie on this curve, write down the values of t and u. **(2)**

(*b*) Make a copy of this diagram and on it sketch the graph of $y = a^{2x}$. **(2)**

(*c*) Find the coordinates of the point of intersection of $y = a^{2x}$ with the line $x = 1$. **(1)**

20. Diagram 1 shows 5 cars travelling up an incline on a roller-coaster. Part of the roller-coaster rail follows the curve with equation $y = 8 + 5\cos\frac{1}{2}x$.

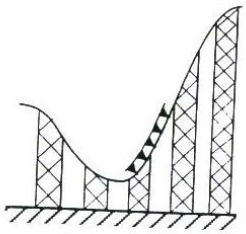


Diagram 1

Diagram 2 shows an enlargement of the last car and its position relative to a suitable set of axes. The floor of the car lies parallel to the tangent at P, the point of contact.

Calculate the acute angle a between the floor of the car and the horizontal when the car is at the point where $x_p = \frac{7\pi}{3}$.

Express your answer in degrees. **(4)**

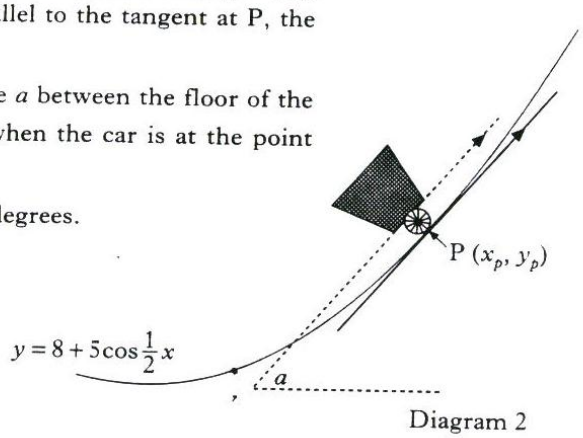


Diagram 2

[END OF QUESTION PAPER]