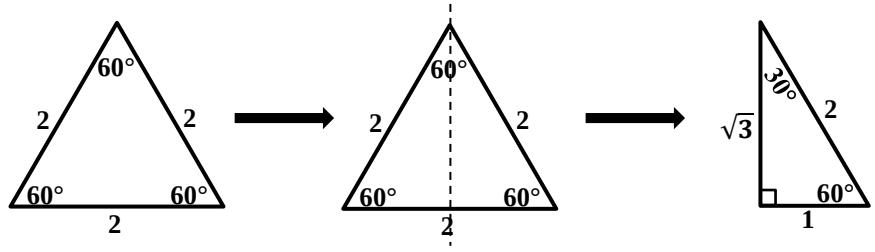
Trig Equations

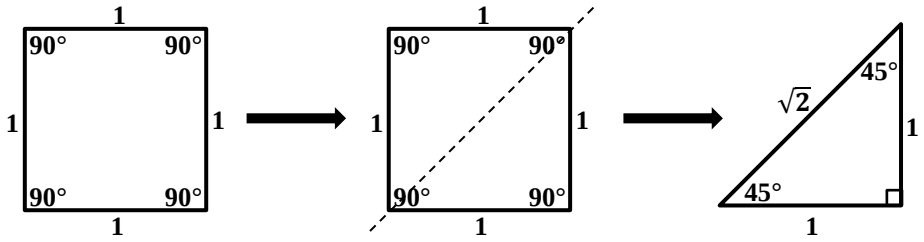
SPTA Mathematics – Higher Notes

Exact Values:

- We saw in National 5 how to solve Trig Equations using a calculator.
- In Higher we can solve Trig Equations without a calculator if we know our 2 **EXACT VALUE TRIANGLES**.
- The first Triangle allows us to find the exact values of 30° & 60° as follows:



- The second Triangle allows us to find the exact value of 45° as follows:



It doesn't matter which 45° you use!!

- The exact values of 0°, 90°, 180°, 270° & 360° should be known from the Trig Graphs.
- This can all be summarized in the following table.

Angle	Sin	Cos	Tan
0°	0	1	0
30°	$^1/_2$	$^{\sqrt{3}}/_2$	$^1/_{\sqrt{3}}$
45°	$^1/_{\sqrt{2}}$	$^1/_{\sqrt{2}}$	1
60°	$^{\sqrt{3}}/_2$	$^1/_2$	$\sqrt{3}$

Angle	Sin	Cos	Tan
90°	1	0	—
180°	0	−1	0
270°	−1	0	—
360°	0	1	0

- Either memorize the Table or the Triangles, (probably easier to memorize the triangles!!!).

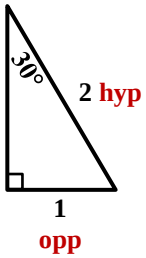
Examples:

1. State the exact values of:

a) sin30°

$sinx° = \frac{opp}{hyp}$

$sin30° = \frac{1}{2}$

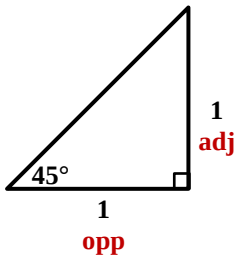


(b) tan45°

$tanx° = \frac{opp}{adj}$

$tan45° = \frac{1}{1}$

$tan45° = 1$



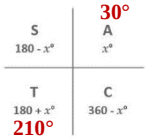
c) cos210°

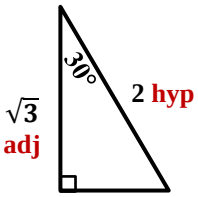
$cos210° = -cos30°$

$cosx° = \frac{adj}{hyp}$

$cos30° = \frac{\sqrt{3}}{2}$

$cos210° = -\frac{\sqrt{3}}{2}$





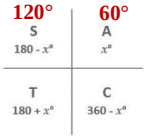
(c) sin120°

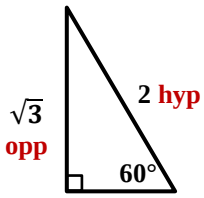
$sin120° = sin60°$

$sinx° = \frac{opp}{hyp}$

$sin60° = \frac{\sqrt{3}}{2}$

$sin120° = \frac{\sqrt{3}}{2}$

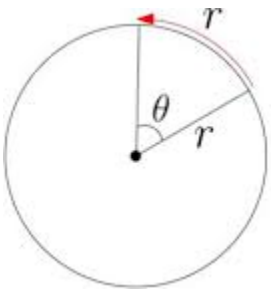




Now attempt Exercise 1 from the Trig Equations booklet

Radians:

- Degrees are not the only unit of measurement for angles, we also use **RADIANS**.
- Radians are based on the ratio: $\frac{Arc\ length}{Radius}$



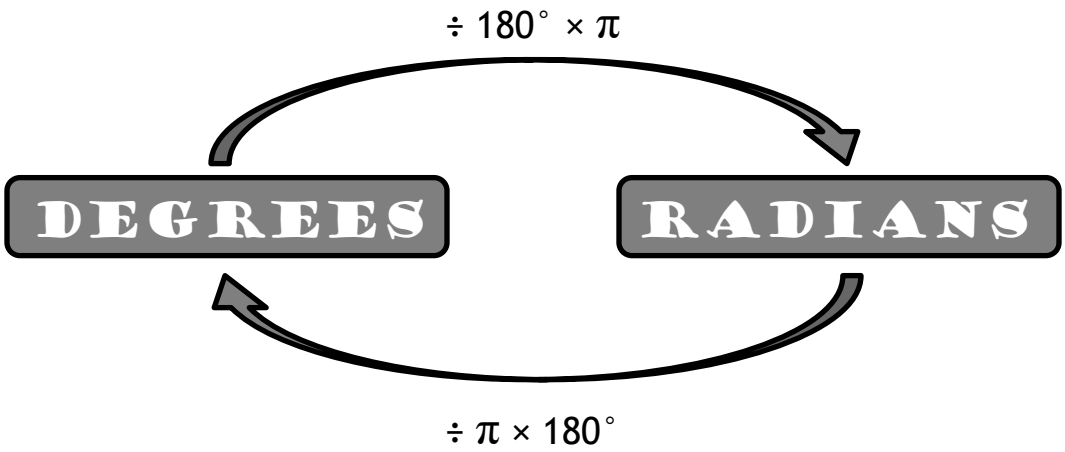
$\theta = 1\ radian$

Circumference of a circle = $2\pi r$

Full circle = 360° $\rightarrow \frac{Arc\ length}{Radius} = \frac{2\pi r}{r} = 2\pi\ radians$

So 360° = 2π radians $\rightarrow$ 180° = π radians

- You must be able to convert between Degrees and Radians as follows:



- When possible we usually write Radians as fractions.

Examples:

2. Convert to Radians:

a) $30° = \frac{30}{180} \times \pi$

$= \frac{\pi}{6}$ radians

(b) $240° = \frac{240}{180} \times \pi$

$= \frac{4\pi}{3}$ radians

3. Convert to Degrees:

a) $\frac{\pi}{3}$ radians $= \frac{\pi}{3} \div \pi \times 180°$

$= 60°$

The π's cancel out!!

(b) $\frac{4\pi}{5}$ radians $= \frac{4\pi}{5} \div \pi \times 180°$

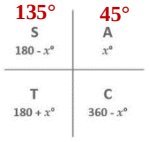
$= 144°$

4. Find the exact values of:

a) $tan\frac{3\pi}{4}$

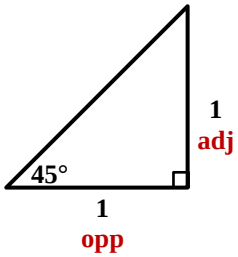
$\frac{3\pi}{4}$ radians $= \frac{3\pi}{4} \div \pi \times 180°$

$= 135°$



$tan135° = -tan45°$

$= -1$

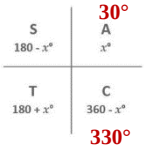


So $tan\frac{3\pi}{4} = -1$

(b) $cos\frac{11\pi}{6}$

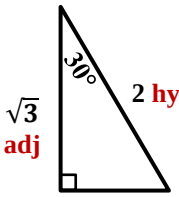
$\frac{11\pi}{6}$ radians $= \frac{11\pi}{6} \div \pi \times 180°$

$= 330°$



$cos330° = cos30°$

$= \frac{\sqrt{3}}{2}$



So $cos\frac{11\pi}{6} = \frac{\sqrt{3}}{2}$

Now attempt Exercise 2 from the Trig Equations booklet

Solving Trig Equations:

- You should know how to solve Trig Equations from National 5
- Remember that there are 2 solutions for each value of x – so $2x$ will give 4 solutions, $3x$ gives 6 …
- We use the CAST diagram to find the second solution.
- Answers can either be in Degrees or Radians, look for a clue in the question!!

Examples:

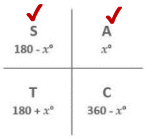
5. Solve for x, $0° \leq x° \leq 360°$

This tells you to answer in degrees – ensure your calculator is in degrees

a) $4\sin x° - 2 = 1$

$4\sin x° = 3$

$\sin x° = \frac{3}{4}$



$x° = sin^{-1}\frac{3}{4}$

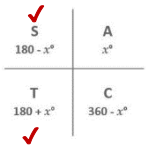
$x° = 48.59°$ or $180° - 48.59°$

$x° = 48.59°$ or $131.41°$

b) $6\cos x° + 5 = 1$

$6\cos x° = -4$

$\cos x° = -\frac{2}{3}$



$x° = cos^{-1}\left(-\frac{2}{3}\right)$

$x° = 180 - 48.19°$ or $180° + 48.19°$

$x° = 131.81°$ or $228.19°$

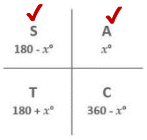
Remember never to type a negative into your calculator when doing trig!!!

6. Solve for x, $0 \le \theta \le 2\pi$ ← This tells you to answer in radians – but keep your calculator in degrees

a) $2\tan\theta + 2 = 3$

$2\tan\theta = 1$

$\tan\theta = \frac{1}{2}$



$\theta = tan^{-1}\frac{1}{2}$

$\theta = 26.57°$ or $180° - 26.57°$

$\theta = 26.57°$ or $153.43°$

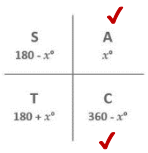
$\theta = 0.464$ or 2.678

Not Exact Values so leave as Decimals

b) $4\cos\theta + 5 = 7$

$4\cos\theta = 2$

$\cos\theta = \frac{1}{2}$

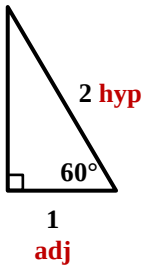


$\theta = cos^{-1}\left(\frac{1}{2}\right)$

$\theta = 60°$ or $360° - 60°$

$\theta = 60°$ or $300°$

$\theta = \frac{\pi}{3}$ or $\frac{5\pi}{3}$



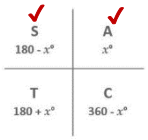
Now attempt Exercise 3 from the Trig Equations booklet

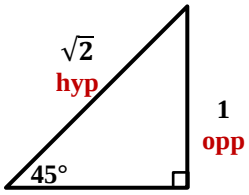
7. Solve for x, $0° \le x° \le 180°$ ← Answer in degrees, but max value is 180°

$\sqrt{2}\sin 3x° - 1 = 0$

$\sqrt{2}\sin 3x° = 1$

$\sin 3x° = \frac{1}{\sqrt{2}}$





$3x° = sin^{-1}\frac{1}{\sqrt{2}}$

Keep adding on 360° to find 6 solutions

$3x° = 45°$ or $180° - 45° \rightarrow 45°$ or $135°$ or $405°$ or $495°$ or $765°$ or $855°$

$x° = 15°$ or $45°$ or $135°$ or $165°$ or ~~$255°$~~ or ~~$285°$~~

$x° = 15°$ or $45°$ or $135°$ or $165°$

Check the range to see what values are valid

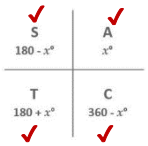
8. Solve for x, $0 \le \theta \le \pi$

$4\cos^2\theta + 2 = 3$

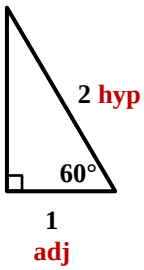
$4\cos^2\theta = 1$

$\cos^2\theta = \frac{1}{4}$

$\cos\theta = \pm\frac{1}{2}$



All 4 quadrants have solutions



$\theta = 60°$ or $180° - 60°$ or $180° + 60°$ or $360° - 60°$

$\theta = 60°$ or $120°$ or ~~$240°$~~ or ~~$300°$~~

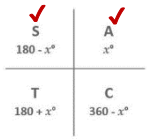
$\theta = \frac{\pi}{3}$ or $\frac{2\pi}{3}$

Check the range to see what values are valid

9. Solve for x, $0° \leq x° \leq 360°$

$5\sin(x° + 45°) = 3$

$\sin(x° + 45°) = \frac{3}{5}$



$x° + 45° = sin^{-1}\frac{3}{5}$

$x° + 45° = 36.87°$ or $180° - 36.87°$

$x° + 45° = 36.87°$ or $143.13°$

$x° = 36.87° - 45°$ or $143.13° - 45°$

$x° = -8.13°$ or $98.13° \rightarrow 351.87°$ or $98.13°$

Can't be left with a negative angle, add 360°

Similarly if you are left with an angle greater than 360° then you would subtract 360°

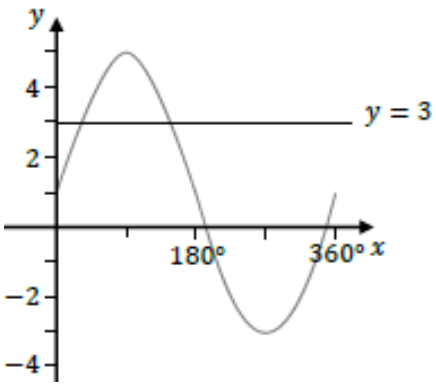
Now attempt Exercise 4 & 5 from the Trig Equations booklet

Intersecting Trig Graphs and Straight Lines:

- The points of intersecting Trig Graphs and Straight Lines can be found by solving the equation produced by making the 2 equations equal.

Examples:

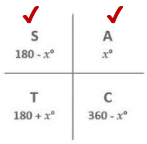
10. Find the coordinates of the points of intersection between the line, $y = 3$, and the trigonometric graph, $y = 4\sin x + 1$, in the range $0 \leq x \leq 360$.
Give your answer as an exact value.



$4\sin x° + 1 = 3$

$4\sin x° = 2$

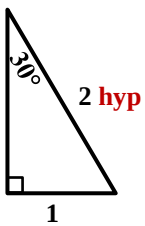
$\sin x° = \frac{1}{2}$



$x° = sin^{-1}\frac{1}{2}$

$x° = 30°$ or $180° - 30°$

$x° = 30°$ or $150°$



So Pts $(30°, 3)$ & $(150°, 3)$

Now attempt Exercise 6 & 7 from the Trig Equations booklet