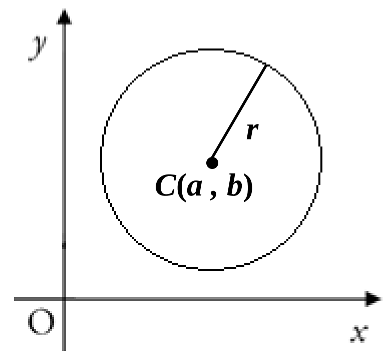
Equation of a Circle:

- To find the equation of a circle you need 2 things:
  - Coordinates of the centre, (a , b)
  - The radius, r



- The equation of a Circle is given by: $(x - a)^2 + (y - b)^2 = r^2$
- The equation of a Circle with centre the origin is given by: $x^2 + y^2 = r^2$
- You will need to remember these two formulae covered in the STRAIGH LINE topic earlier:
  - Distance Formula: $\sqrt{(x_B - x_A)^2 + (y_B - y_A)^2}$
  - Midpoint Formula: $\left(\frac{x_A+x_B}{2} , \frac{y_A+y_B}{2}\right)$

You do not need to expand the brackets unless the question specifies to do so but you must always square the radius!!

Must know!!

- **<u>CONGRUENT</u>** – Objects which are the **<u>EXACT</u>** same size and shape.

Examples:

1. Write down the equation of the circle with:

a) Centre (0 , 0) and radius = 6:

$$x^2 + y^2 = 6^2$$
$$\Rightarrow \quad x^2 + y^2 = 36$$

b) Centre (5 , –2) and radius = 3.5:

$$(x - 5)^2 + (y - (-2))^2 = 3.5^2$$
$$\Rightarrow \quad (x - 5)^2 + (y + 2)^2 = 12.25$$

c) Centre (3 , 0) and radius = $4\sqrt{3}$:

$$(x - 3)^2 + (y - 0)^2 = (4\sqrt{3})^2$$
$$\Rightarrow \quad (x - 3)^2 + y^2 = (\sqrt{48})^2$$
$$\Rightarrow \quad (x - 3)^2 + y^2 = 48$$

d) Centre (3 , 6) passing through (–1 , 4) :

$$\text{Radius} = \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2}$$

$$\text{Radius} = \sqrt{(-1-3)^2 + (4-6)^2}$$

$$\text{Radius} = \sqrt{(-4)^2 + (-2)^2}$$

$$\text{Radius} = \sqrt{16+4}$$

$$\text{Radius} = \sqrt{20}$$

No need to square root it as going to square it for the equation!!

so $(x-3)^2 + (y-6)^2 = (\sqrt{20})^2$

$$(x-3)^2 + (y-6)^2 = 20$$

2. State the Centre and Radius of these circles:

a) $(x-8)^2 + (y+3)^2 = 36$ Centre (8 , –3) & Radius = $\sqrt{36} = 6$

b) $x^2 + y^2 = 144$ Centre (0 , 0) & Radius = $\sqrt{144} = 12$

c) $(x+7)^2 + (y-4)^2 = 18$ Centre (–7 , 4) & Radius = $\sqrt{18} = 3\sqrt{2}$

Normally leave as a SURD

3. Find the equation of the circle when the points *A*(-5 , 3) and *B*(3 , 1) are the endpoints of the diameter.

$$\text{Midpoint } AB = \left(\frac{x_A+x_B}{2}, \frac{y_A+y_B}{2}\right)$$

$$\text{Midpoint } AB = \left(\frac{-5+3}{2}, \frac{3+1}{2}\right)$$

$$\text{Midpoint } AB = (-1, 2)$$

$$\text{Radius} = \sqrt{(x_M - x_A)^2 + (y_M - y_A)^2}$$

$$\text{Radius} = \sqrt{(-1-(-5))^2 + (2-3)^2}$$

$$\text{Radius} = \sqrt{4^2 + (-1)^2}$$

$$\text{Radius} = \sqrt{16+1}$$

$$\text{Radius} = \sqrt{17}$$

so $(x-(-1))^2 + (y-2)^2 = (\sqrt{17})^2$

$$(x+1)^2 + (y-2)^2 = 17$$

4. a) Prove that the triangle *ABC* is right angled when *A*(–3 , –2) , *B*(3 , –1) and *C*(2 , 5),

$AB = \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2}$

$AB = \sqrt{(3-(-3))^2 + (-1-(-2))^2}$

$AB = \sqrt{6^2 + 1^2}$

$AB = \sqrt{36 + 1}$

$AB = \sqrt{37}$

Could use gradients instead

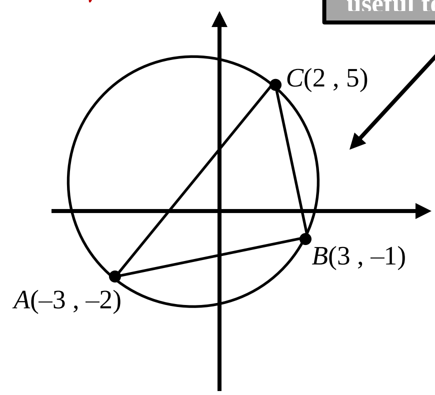
$AC = \sqrt{(2-(-3))^2 + (5-(-2))^2}$

$AC = \sqrt{5^2 + 7^2}$

$AC = \sqrt{25 + 49}$

$AC = \sqrt{74}$

If no diagram is given it may be useful to sketch one!



$BC = \sqrt{(2-3)^2 + (5-(-1))^2}$

$BC = \sqrt{(-1)^2 + 6^2}$

$BC = \sqrt{1 + 36}$

$BC = \sqrt{37}$

Since $AC^2 = AB^2 + BC^2$ the triangle is right angled.

Angle in a semicircle from Nat 5

b) Hence find the equation of the circle passing through these 3 points.

From part (a), we can see that *AC* is the diameter of the circle.

Midpoint $AC = \left(\frac{x_A + x_C}{2}, \frac{y_A + y_C}{2}\right)$

Midpoint $AC = \left(\frac{-3+2}{2}, \frac{-2+5}{2}\right)$

Midpoint $AC = \left(-\frac{2}{2}, \frac{3}{2}\right)$

Radius $= \sqrt{(x_M - x_A)^2 + (y_M - y_A)^2}$

Radius $= \sqrt{(-1-(-5))^2 + (2-3)^2}$

Radius $= \sqrt{4^2 + (-1)^2}$

Radius $= \sqrt{16 + 1}$

Radius $= \sqrt{17}$

so $(x-(-1))^2 + (y-2)^2 = (\sqrt{17})^2$

$(x+1)^2 + (y-2)^2 = 17$

Now attempt Exercise 1 from the Circles booklet

General Equation of a Circle:

- The equation of a circle can also be given in the following form (earlier equation expanded!!):

  $x^2 + y^2 + 2gx + 2fy + c = 0$ where the centre is $(-g\ ,-f)$ and the radius is $\sqrt{g^2 + f^2 - c}$

- As soon as you see an equation of a circle in a question it is **ALWAYS** worthwhile stating the centre & radius, regardless of what format the equation is given in or what the question is asking!!

Examples:

5. Find the radius and centre of: $x^2 + y^2 + 4x - 8y + 7 = 0$

$2g = 4$ $2f = -8$ $c = 7$

$g = 2$ $f = -4$ so centre is $(-2\ , 4)$

Radius $= \sqrt{g^2 + f^2 - c}$

Radius $= \sqrt{2^2 + (-4)^2 - 7}$

You should use g and f NOT the centre

Radius $= \sqrt{4 + 16 - 7}$

Radius $= \sqrt{13}$ units

6. Find the radius and centre of: $2x^2 + 2y^2 - 6x + 10y - 2 = 0$

$x^2 + y^2 - 3x + 5y - 1 = 0$

The equation must be in the form: $x^2 + y^2 + 2gx + 2fy + c = 0$ so divide the original by 2

$2g = -3$ $2f = 5$ $c = -1$

$g = \frac{-3}{2}$ $f = \frac{5}{2}$

so centre is $\left(\frac{3}{2}\ , \frac{-5}{2}\right)$

Radius $= \sqrt{g^2 + f^2 - c}$

Radius $= \sqrt{\left(\frac{-3}{2}\right)^2 + \left(\frac{5}{2}\right)^2 - (-1)}$

Radius $= \sqrt{\frac{9}{4} + \frac{25}{4} + 1}$

Radius $= \sqrt{\frac{38}{4}} = \frac{\sqrt{38}}{2}$ units

7. Explain why $x^2 + y^2 - 4x + 2y + 42 = 0$ is not an equation of a circle.

$2g = -4 \qquad 2f = 2 \qquad c = 42$
$g = -2 \qquad f = 1$

so centre is (2 , –1)

Did not require the centre for this question.

$\text{Radius} = \sqrt{g^2 + f^2 - c}$

$\text{Radius} = \sqrt{(-2)^2 + 1^2 - 42}$

$\text{Radius} = \sqrt{4 + 1 - 42}$

$\text{Radius} = \sqrt{-37}$

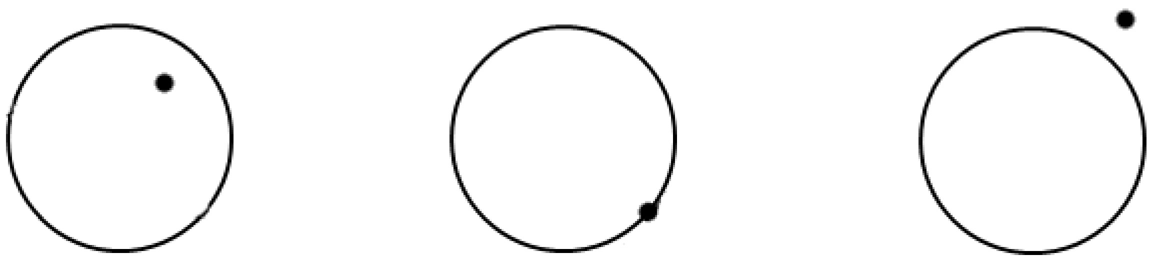
since root of a negative is not possible, the equation above does not represent a circle.

This is not totally accurate, but at higher it is fine!!

Now attempt Exercise 2 from the Circles booklet

Points Inside/Outside/On Circles:

- A point can lie INSIDE a Circle, ON a Circle or OUTSIDE a Circle as follows:



INSIDE the circle ON the ircle OUTSIDE the circle

- To determine where a point lies in relation to the circle we compare the distance from the CENTRE of the circle to the point with the RADIUS of the Circle.
  - Distance = Radius → Point lies **ON** the Circle.
  - Distance > Radius → Point lies **OUTSIDE** the Circle.
  - Distance < Radius → Point lies **INSIDE** the Circle.
- If asked to PROVE or SHOW that a point lies **ON** the circle you could sub the coordinate into the equation instead to get zero if it does lie on the circle (See example 14(a) later on).

Examples:

8. Where does each point below lie in relation to the Circle given:

a) $(x-8)^2+(y+3)^2=36$ Pt A(3 , −10)

Centre (8 , –3) & Radius = $\sqrt{36}$ = 6

$AC=\sqrt{(8-3)^2+(-3-(-10))^2}$

$AC=\sqrt{5^2+7^2}$

$AC=\sqrt{25+49}$

$AC=\sqrt{74}\approx 8.6$ So Point A lies OUTSIDE the Circle since 8.6 > 6

b) $(x+5)^2+(y-2)^2=117$ Pt B(1 , −4)

Centre (–5 , 2) & Radius = $\sqrt{117}\approx 10.8$

$BC=\sqrt{(-5-1)^2+(2-(-4))^2}$

$BC=\sqrt{(-6)^2+6^2}$

$BC=\sqrt{36+36}$

$BC=\sqrt{72}\approx 8.5$ So Point B lies INSIDE the Circle since $\sqrt{72}<\sqrt{117}$

c) $x^2+y^2+4x-8y+7=0$ Pt D(1 , 2)

$2g=4$ $2f=-8$ $c=7$

$g=2$ $f=-4$ so centre is (–2 , 4)

Radius $=\sqrt{g^2+f^2-c}$

Radius $=\sqrt{2^2+(-4)^2-7}$

Radius $=\sqrt{4+16-7}$

Radius $=\sqrt{13}\approx 3.6$

$BC=\sqrt{(-2-1)^2+(4-2)^2}$

$BC=\sqrt{(-3)^2+2^2}$

$BC=\sqrt{9+4}$

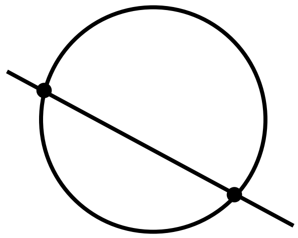
$BC=\sqrt{13}$

So Point D lies ON the Circle since $\sqrt{13}=\sqrt{13}$

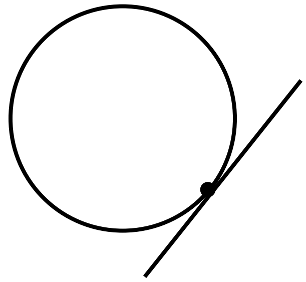
Now attempt Exercise 3 from the Circles booklet

Intersection of Circles with Lines:

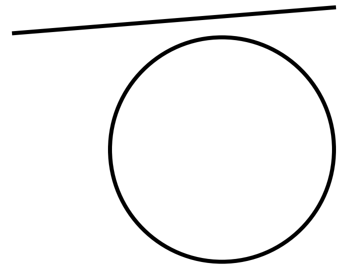
- Straight lines can intersect (meet) circles at either 2, 1 or zero points as follows:



2 Points of intersection
$b^2 - 4ac > 0$



1 Point of intersection
So Tangent to the circle
$b^2 - 4ac = 0$



No Points of intersection
$b^2 - 4ac < 0$

Discriminant from National 5

- To determine the nature of the points of intersection we use the SUBSTITUTION method of Simultaneous equations from Naionalt 5.

Examples:

9. Find where the line $y = 4x$ meets the circle with equation $x^2 + y^2 = 34$

Substitute $y = 4x$ into the circle $x^2 + y^2 = 24 \implies x^2 + (4x)^2 = 34$

$$\implies x^2 + 16x^2 = 34$$

$$\implies 17x^2 = 34$$

$$\implies x^2 = 2$$

$$\implies x = \pm\sqrt{2}$$

When$x = \pm\sqrt{2}$: $y = \pm 4\sqrt{2}$ So Points of Intersection are: $(\sqrt{2}, 4\sqrt{2})$ and $(-\sqrt{2}, -4\sqrt{2})$

10. Find the points where the line $2x + y - 6 = 0$ intersects the circle $x^2 + y^2 - 2x + 2y - 8 = 0$

$2x + y - 6 = 0 \implies y = -2x + 6$ Substitute $y = -2x + 6$ into the circle :

$$x^2 + y^2 - 2x + 2y + 8 = 0 \implies x^2 + (-2x + 6)^2 - 2x + 2(-2x + 6) - 8 = 0$$

$$\implies x^2 + (4x^2 - 24x + 36)^2 - 2x - 4x + 12 - 8 = 0$$

$$\implies 5x^2 - 30x + 40 = 0$$

$$\implies x^2 - 6x + 8 = 0$$

$$\implies (x - 2)(x - 4) = 0$$

$$\implies \mathrm{x} - 2 = 0 \quad x - 4 = 0$$

$$\implies x = 2,\ 4$$

When $x = 2$: $y = -2(2) + 6 \implies y = 2$ so Pt (2 , 2)

When $x = 4$: $y = -2(4) + 6 \implies y = -2$ so Pt (4 , –2)

11. Prove that the line $x + y = 4$ is a tangent to the circle $x^2 + y^2 + 6x + 2y - 22 = 0$ and find the point of contact.

$x + y = 4 \implies y = -x + 4$ Substitute $y = -x + 4$ into the circle :

$$x^2 + y^2 + 6x + 2y - 22 = 0 \implies x^2 + (-x + 4)^2 + 6x + 2(-x + 4) - 22 = 0$$

$$\implies x^2 + x^2 - 8x + 16 + 6x - 2x + 8 - 22 = 0$$

The Discriminant could be used at this point as shown below →

$$\implies 2x^2 - 4x + 2 = 0$$

$$\implies x^2 - 2x + 1 = 0$$

$$\implies (x - 1)(x - 1) = 0$$

$$\implies x - 1 = 0$$

$$\implies x = 1$$

When $x = 1$: $y = -1 + 4 \implies y = 3$ so Pt (1 , 3)

Since only one point of contact the line is a tangent to the circle at the point (1 , 3)

OR using the **DISCRIMINANT**:

$x^2 - 2x + 1 = 0$ so $a = 1,\ b = -2,\ c = 1$

$b^2 - 4ac = (-2)^2 - 4(1)(1)$

$= 4 - 4$

$= 0$ since $b^2 - 4ac = 0$ the line is a tangent to the circle.

$x^2 - 2x + 1 = 0$

$\Longrightarrow\ (x-1)(x-1) = 0$

$\Longrightarrow\ x - 1 = 0$

$\Longrightarrow\ x = 1$

When $x = 1$: $y = -1 + 4 \Longrightarrow y = 3$ so Pt $(1\ ,3)$

In example 11 above it is probably more sensible to use the first method as you need to find the Point of Intersection anyway. If only asked to prove the Line is a Tangent then either method is suitable.

12. Determine whether the line $x + y = 4$ intersects the circle $x^2 + y^2 + 2x - 3y + 2 = 0$

$x - y = 5 \Longrightarrow x = y + 5$ Substitute $x = y + 5$ into the circle :

Can change to $x =$ or $y =$

$x^2 + y^2 + 2x - 3y + 2 = 0 \Longrightarrow (y + 5)^2 + y^2 + 2(y + 5) - 3y + 2 = 0$

$\Longrightarrow y^2 + 10y + 25 + y^2 + 2y + 10 - 3y + 2 = 0$

$\Longrightarrow 2y^2 + 9y + 37 = 0$

$2y^2 + 9y + 37 = 0$ so $a = 2,\ b = 9,\ c = 37$

$b^2 - 4ac = 9^2 - 4(2)(37)$

$= 81 - 296$

$= -215$ since $b^2 - 4ac < 0$ the line does NOT intersect the circle.

13. The line with equation $x - 3y = k$ is a tangent to the circle $x^2 + y^2 - 6x + 8y + 15 = 0$
Find the possible values of k

Change to $x =$ to avoid fractions

$x - 3y = k \implies x = 3y + k$ Substitute $x = 3y + k$ into the circle :

$$x^2 + y^2 - 6x + 8y + 15 = 0 \implies (3y + k)^2 + y^2 - 6(3y + k) + 8y + 15 = 0$$
$$\implies 9y^2 + 6ky + k^2 + y^2 - 18y - 6k + 8y + 15 = 0$$
$$\implies 10y^2 + 6ky - 10y + k^2 - 6k + 15 = 0$$
$$\implies 10y^2 + (6k - 10)y + (k^2 - 6k + 15) = 0$$

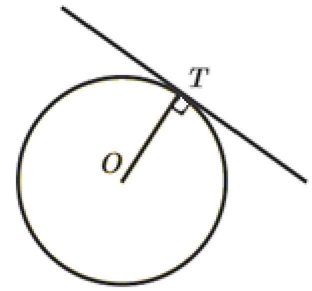
so $a = 10,\ b = 6k - 10,\ c = k^2 - 6k + 15$

Since the line is a tangent $b^2 - 4ac = 0$

$$\implies (6k - 10)^2 - 4(10)(k^2 - 6k + 15) = 0$$
$$\implies 36k^2 - 120k + 100 - 40k^2 + 240k - 600 = 0$$
$$\implies -4k^2 + 120k - 500 = 0$$
$$\implies k^2 - 30k + 125 = 0$$
$$\implies (k - 5)(k - 25) = 0$$
$$\implies k - 5 = 0 \quad k - 25 = 0$$
$$\implies k = 5,\ 25$$

Now attempt Exercise 4 from the Circles booklet

<u>Equation of a Tangent to a Circle:</u>



- You can find the equation of a tangent to a circle if you know the point of contact as follows:
    - Find the centre of the circle
    - Find the gradient of the radius to the point of contact, m_{RAD},
    - Find the perpendicular gradient for the tangent using, $m_{TAN} \times m_{RAD} = -1$
    - Use $y - b = m(x - a)$ to find the tangent's equation.

- Remember: **<u>CONGRUENT</u>** – Objects which are the **<u>EXACT</u>** same size and shape.

<u>Examples:</u>

Use substitution for this one!!

14. a) Prove that the point $A(2\,,-1)$ lies on the circle $x^2 + y^2 + 8x - 3y - 24 = 0$

Sub the point into the circle equation: $2^2 + (-1)^2 + 8(2) - 3(-1) - 24$

$= 4 + 1 + 16 + 3 - 24$

$= 0$ hence point lies on the circle.

b) Find the equation of the tangent to the circle at this point.

$2g = 8$ $\quad 2f = -3$ $\quad c = -24$

$g = 4$ $\quad f = -{}^{3}/_{2}$ $\quad$ so centre is $C(-4\,,{}^{3}/_{2})$

$$\text{Radius} = \sqrt{g^2 + f^2 - c}$$

$$\text{Radius} = \sqrt{4^2 + \left(\frac{-3}{2}\right)^2 + 24}$$

$$\text{Radius} = \sqrt{16 + \frac{9}{4} + 24}$$

$$\text{Radius} = \sqrt{42\tfrac{1}{4}}$$

Did not require the radius for this question.

$$m_{AC} = \frac{y_C - y_A}{x_C - x_A} = \frac{-4-2}{{}^{3}/_{2} - (-1)} = \frac{-6}{{}^{5}/_{2}} = -\frac{12}{5}$$

Since perpendicular

$m_{Rad} \times m_{Tan} = -1$

so $m_{Tan} = {}^{5}/_{12}$

$y - b = m(x - a)$

$y - 2 = {}^{5}/_{12}(x - (-1))$

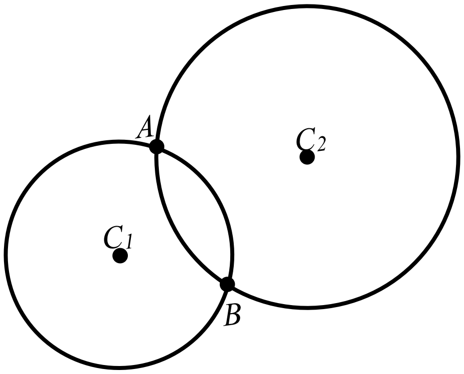
$12y - 24 = 5x + 5$

$12y = 5x + 29$

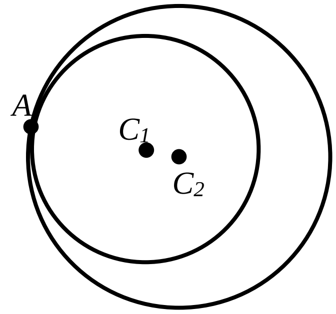
Now attempt Exercise 5 from the Circles booklet

Intersection of Circles:

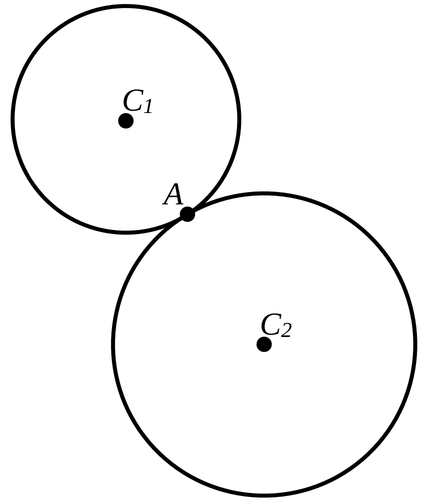
- 2 Circles can touch or intersect in 3 ways:



2 Points of intersection
Circles cross over.
Distance between centres is
less than sum of the 2 Radii.



1 Point of intersection
Circles touch **INTERNALLY**
Distance between centres
equals difference of the 2 Radii.



1 Point of intersection
Circles touch **EXTERNALLY**
Distance between centres
equals sum of the 2 Radii.

Examples:

15. Circle *P* has centre (–4 , –1) and radius 2 units.

Circle Q has equation $x^2 + y^2 - 2x + 6y + 1 = 0$.

Show that circles *P* and *Q* do not touch.

$2g = -2$ $2f = 6$ $c = 1$

$g = -1$ $f = 3$ so centre is $C(1, -3)$

Radius $= \sqrt{g^2 + f^2 - c}$

Radius $= \sqrt{(-1)^2 + 3^2 - 1}$

Radius $= \sqrt{9} = 3$

Centres $= \sqrt{(x_Q - x_P)^2 + (y_Q - y_P)^2}$

Centres $= \sqrt{(1-(-4))^2 + (-3-(-1))^2}$

Centres $= \sqrt{5^2 + (-2)^2}$

Centres $= \sqrt{25 + 4} = \sqrt{29}$

Since $R_1 + R_2 = 3 + 2 = 5 < \sqrt{29}$ the circles do **NOT** touch.

16. Circle R has equation $x^2 + y^2 - 2x - 4y - 4 = 0$ and

Circle S has equation $(x-4)^2 + (y-6)^2 = 4$

Show that circles R and S touch externally.

$2g = -2 \quad 2f = -4 \quad c = -4$

$g = -1 \quad f = -2$

so centre is $C_R(1, 2)$

Radius $= \sqrt{g^2 + f^2 - c}$

Radius $= \sqrt{(-1)^2 + (-2)^2 + 4}$

Radius $= \sqrt{9} = 3$

Centre is $C_S(4, 6)$

Radius $= \sqrt{4}$

Radius $= 2$

Centres $= \sqrt{(x_S - x_R)^2 + (y_S - y_R)^2}$

Centres $= \sqrt{(4-1)^2 + (6-2)^2}$

Centres $= \sqrt{3^2 + 4^2}$

Centres $= \sqrt{9 + 16} = \sqrt{25} = 5$

Since $R_1 + R_2 = 3 + 2 = 5$ the circles touch **EXTERNALLY**.

17. Circle T has equation $x^2 + y^2 - 6x + 6y - 46 = 0$ and

Circle U has equation $(x+1)^2 + y^2 = 9$

Show that circles T and U touch internally.

$2g = -6 \quad 2f = 6 \quad c = -46$

$g = -3 \quad f = 3$

so centre is $C_R(3, -3)$

Radius $= \sqrt{g^2 + f^2 - c}$

Radius $= \sqrt{(-3)^2 + 3^2 + 46}$

Radius $= \sqrt{64} = 8$

Centre is $C_S(-1, 0)$

Radius $= \sqrt{9}$

Radius $= 3$

Centres $= \sqrt{(x_S - x_R)^2 + (y_S - y_R)^2}$

Centres $= \sqrt{(3 - (-1)^2 + (-3 - 0)^2}$

Centres $= \sqrt{4^2 + (-3)^2}$

Centres $= \sqrt{16 + 9} = \sqrt{25} = 5$

Since $R_1 - R_2 = 8 + 3 = 5$ the circles touch **INTERNALLY.**

Now attempt Exercise 6 from the Circles booklet

Circle Problems:

- You are sometimes asked to find points on the Circles or centres/radii of adjoining circles:
- You have all the knowledge to solve these problems, it is simply knowing how to apply it!!
- Some of these can be solved more efficiently using **VECTORS**, the next topic we will look at.

Examples:

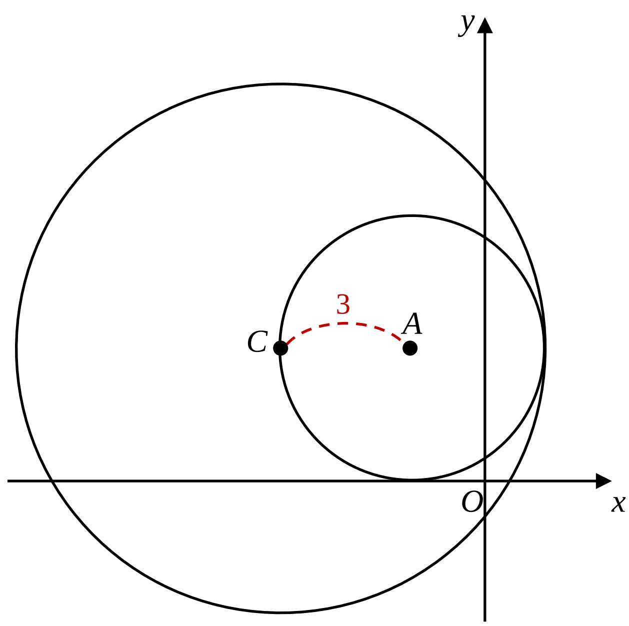
18. A circle, centre *C*, has the equation

$x^2 + y^2 + 16x - 6y + 37 = 0$

A smaller circle, centre *A*, touches the bigger circle internally and passes through *C*.

The centres of the circles lie on a line parallel with the x-axis.

Find the equation of the circle, centre *A*.



$2g = 16 \quad 2f = -6 \quad c = 37$

$g = 8 \quad f = -3$

so centre is $C(-8, 3)$

$\text{Radius} = \sqrt{g^2 + f^2 - c}$

$\text{Radius} = \sqrt{8^2 + (-3)^2 + 37}$

$\text{Radius} = \sqrt{36} = 6$ so the Radius of the smaller circle is 3

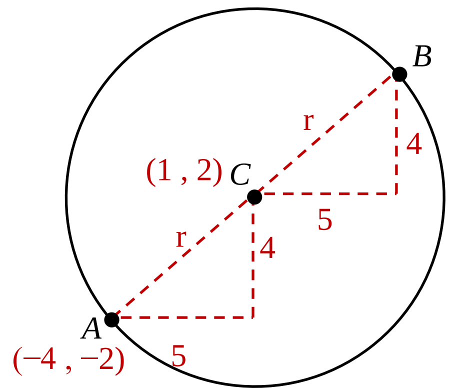
So the Centre *A* is $(-5, 3)$ giving $(x+5)^2 + (y-3)^2 = 9$ ← **No need to expand the brackets**

19. A circle, centre C, has the equation

$x^2 + y^2 - 2x - 4y - 36 = 0$

The point A $(-4, -2)$ lies on the circle.

Find the point B, which lies diametrically opposite point A.



$2g = -2$ $2f = -4$ $c = -36$

$g = -1$ $f = -2$

so centre is $C\,(1, 2)$

Ratio $AC : CB$ is $1 : 1$

So the point B is $(6, 6)$

Diametrically opposite means a line can join the 2 points passing through the Centre

This method is known as the STEP OUT method.

20. The smaller circle with Centre A has equation,

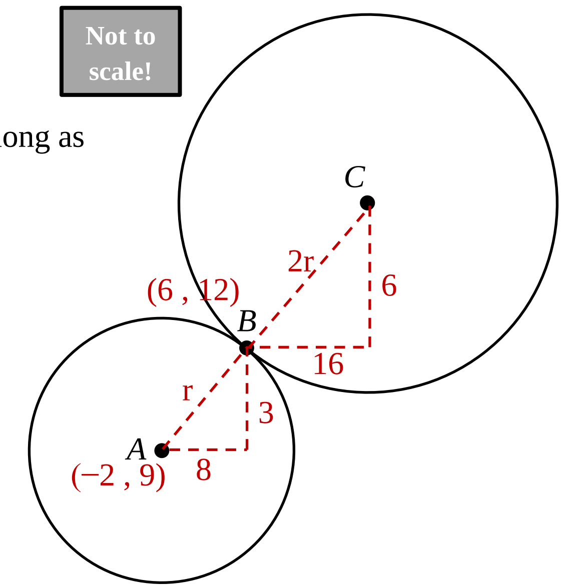
$x^2 + y^2 + 4x - 18y + 12 = 0$

The radius of the larger circle, centre C, is twice as long as the radius of the smaller circle.

The coordinates of B, the point where the 2 circles meet externally is $B(6, 12)$.

What is the equation of the larger circle?

Not to scale!



$2g = 4$ $2f = -18$ $c = 12$

$g = 2$ $f = -9$

so centre is $A\,(-2, 9)$

Ratio $AB : BC$ is $1 : 2$

So the point C is $(22, 18)$

You may find it easier to do this part using VECTORS which we will see next topic!!

Radius $= \sqrt{g^2 + f^2 - c}$

Radius $= \sqrt{2^2 + (-9)^2 - 12}$

Radius $= \sqrt{73}$ so the Radius of the larger circle is $2\sqrt{73}$

Equation of the larger circle is $(x - 22)^2 + (y - 18)^2 = 292$

Again no need to expand the brackets

Now attempt Exercise 7A/B – 9 from the Circles booklet